are determined by the form coefficients of the considered grid, which depend only on the latter geometric properties.

The properties of Green's functions and of perturbation potentials for grids of oscillating mono- and dipoles have been investigated.

The author thanks M. I. Gurevich for discussing certain results of this investigation and for his valuable advice.

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THE FORM OF FRESH-WATER LENS FOR LINEAR EVAPORATION LAW

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Solution of the problem of the stabilized lens of fresh-water filtering from a channel is derived. At the free surface of the latter the stream function is specified in the form of a linear combination of coordinates which includes the particular relationships previously considered by Emikh [1]. The boundary separating fresh and saline waters, the free surface, and the characteristic dimensions of the lens are determined with the use of the analytic theory of linear differential equations.

1. Statement of the problem. The geometry of the considered flow region is shown in Fig. 1. A porous medium of constant porosity m and filtration coefficient K occupies the lower half-plane $y \leqslant 0$. Fresh water of density ρ_1 filters from the channel $A'BA$ of width $2h$, and penetrates the surface of the more sense ground water depressing it in the form of a lens $G \cup G'$. It is assumed that the saline water of density ρ_2 $(\rho_2 > \rho_1)$ lying below the separation

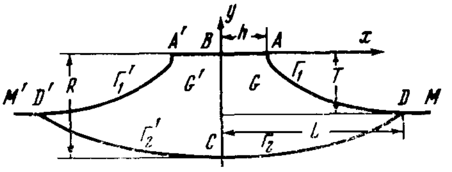

Fig. 1

boundary $M'D'CDM$ is stagnant and that the motion in the lens is stabilized, owing to the evaporation from the free surface $\Gamma_1 \cup \Gamma_1'$ which is specified by the following linear relation between the stream function ψ and the free surface coordinates: $\psi - \varepsilon_2 y + \varepsilon_1 x = \text{const}$. Owing to the flow symmetry about the axis $x = 0$, only its right-hand half $x \geqslant 0$ is considered below.

By introducing parameters

$$\delta = \frac{\rho_2}{\rho_1} - 1, \quad \delta_1 = T\frac{\rho_2}{\rho_1}, \quad \mu = \frac{\varepsilon_1}{K}, \quad \nu = \frac{\varepsilon_2}{K}$$

the reduced potential φ and the reduced stream function defined as previously by ψ, being harmonic functions, for the assumptions made above, satisfy the following conditions [2]:

$$\varphi\,|_{AB} = \psi\,|_{BC} = \psi\,|_{CD} = (\varphi - \delta y - \delta_1)\,|_{CD} = (\varphi + y)\,|_{DA} = (\psi + \mu x - \nu y - \mu L - \nu T)_{DA} = 0 \tag{1.1}$$

In the process of solving this problem parameters μ, ν, the channel width $2h$, and the maximum distance T of the free surface from the axis $y = 0$ are considered to be specified positive numbers and

$$0 < \mu, \nu < 1, \qquad \mu < \delta \tag{1.2}$$

We introduce the following functions: $z(\zeta)$ $(z = x + iy)$ which conformally maps the upper half-plane R_ζ^+ of the plane of the auxiliary variable $\zeta = \xi + i\eta$ onto the flow region G (the correspondence of points is shown in Fig. 2) and

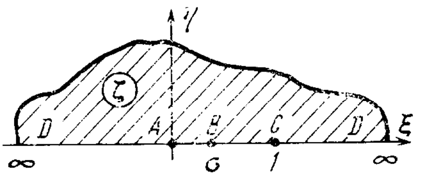

Fig. 2

$$\omega = \varphi + i\psi, \qquad w = \frac{d\omega}{dz} = u + iv \tag{1.3}$$

$$Z = i\sqrt{\zeta - \sigma}\,\frac{dz}{d\zeta}, \qquad \Omega = \sqrt{\zeta - \sigma}\,\frac{d\omega}{d\zeta}$$

where $\sqrt{\zeta - \sigma}$ denotes a branch continuous in R_ζ^+ such that $\arg\sqrt{\zeta - \sigma} = 0$ when $\zeta = \xi > \sigma$.

2. Construction of functions Ω and Z. With the indicated selection of the root branch, the conditions (1.1) lead to the following boundary value problem for Ω and Z (see [2]): along the real axis $\eta = 0$ we have

$$\begin{aligned} &\operatorname{Im}(\Omega - Z) = \operatorname{Im}[(i - \nu)\,\Omega + i\mu Z] = 0 \quad (-\infty < \xi < 0)\\ &\operatorname{Im}\Omega = \operatorname{Im} Z = 0 \quad (0 < \xi < 1)\\ &\operatorname{Im}\Omega = \operatorname{Im}(i\Omega + i\delta Z) = 0 \quad (1 < \xi < +\infty)\end{aligned} \tag{2.1}$$

($\operatorname{Im} f$ and $\operatorname{Re} f$ are the imaginary and the real parts of f, respectively). Boundary conditions (2.1) and the Schwarz principle of symmetry make it possible to extend Ω and Z into the lower half-plane and, then, to determine formulas for bypassing singular points 0, 1 and ∞ [2]. It can be shown that in this case

$$\begin{aligned} &[\nu - i(1 + \mu)]\,\Omega^- = [\nu + i(1 - \mu)]\,\Omega^+ + 2i\mu Z^+ \quad (\zeta = \zeta_1 = 0)\\ &[\nu - i(1 + \mu)]\,Z^- = 2i\Omega^+ + [\nu - i(1 - \mu)]\,Z^+\\ &\Omega^- = \Omega^+, \quad \delta Z^- = -2\Omega^+ - \delta Z^+ \quad (\zeta = \zeta_2 = 1)\\ &[\nu + i(1 + \mu)]\,\Omega^- = [\nu - i(1 - \mu)]\,\Omega^+ - 2i\mu Z^+ \quad (\zeta = \zeta_3 = \infty)\\ &[\nu + i(1 + \mu)]\,Z^- = -2\,[\nu - i(\delta + 1 - \mu)]\,\Omega^+ / \delta - [\nu + i(1 - \mu - 4\mu/\delta)]\,Z^+\end{aligned} \tag{2.2}$$

where for $k = 1,2$ symbols Ω^+ and Z^+ denote, respectively, the limit values of Ω and Z from R_ζ^+ along segment (ζ_k, ζ_{k+1}) of the real axis, and Ω^- and Z^- denote the limit values of these functions along the same segment after point ζ_k had been bypassed in a clockwise direction by a 2π angle. For ζ_3 symbols $\Omega^\pm$ and $Z^\pm$ have the same meaning but with respect to the variable $\tau = -1/\zeta$.

Let us consider the Riemann differential equation

$$\frac{d^2\Phi}{d\zeta^2} + k_1(\zeta)\frac{d\Phi}{d\zeta} + k_2(\zeta)\Phi = 0$$
$$k_1(\zeta) = \frac{1-\alpha_1-\beta_1}{\zeta} + \frac{1-\alpha_2-\beta_2}{\zeta-1} \tag{2.3}$$
$$k_2(\zeta) = \frac{1}{\zeta(\zeta-1)}\left(-\frac{\alpha_1\beta_1}{\zeta} + \frac{\alpha_2\beta_2}{\zeta-1} + \alpha_3\beta_3\right)$$

with three singular points 0, 1 and ∞ with exponents $\alpha_1, \beta_1, \ldots, \beta_3$ which satisfy the Fuchs condition [3]

$$\alpha_1 + \beta_1 + \alpha_2 + \beta_2 + \alpha_3 + \beta_3 = 1 \tag{2.4}$$

We assume that for a certain choice of exponents the unknown functions Ω and Z are solutions of this equation [2]. To determine these we make use of the fact that, except in certain particular cases, the fundamental solutions of Eq. (2.3) in the neighborhood of the singular point ζ_k $(k = 1, 2)$ can be represented in the form $\Phi_1^k(\zeta) = (\zeta - \zeta_k)^{\alpha_k}\Phi_{1k}(\zeta)$ and $\Phi_2^k(\zeta) = (\zeta - \zeta_k)^{\beta_k}\Phi_{2k}(\zeta)$, where functions Φ_{1k} and Φ_{2k} are nonzero at point ζ_k and holomorphic in its neighborhood. In the neighborhood of the infinitely distant singular point these representations remain valid but with the obvious substitution of. $-1/\zeta$ for ζ, and α_3 and β_3 for α_k and β_k, respectively. By virtue of the above assumption it must be possible to find such constants a_k, b_k, c_k and d_k that in some neighborhood of point ζ_k

$$\Omega = a_k\Phi_1^k + b_k\Phi_2^k, \qquad Z = c_k\Phi_1^k + d_k\Phi_2^k \tag{2.5}$$

This makes it possible to determine formulas for bypassing Ω and Z. Comparing these with (2.2) and using the reasonable limitation $w \not\equiv \text{const}$, we obtain

$$\alpha_1 = m_1, \qquad \beta_1 = 2p + n_1, \qquad \alpha_2 = m_2$$
$$\beta_2 = n_2 + {}^1/_2, \qquad \alpha_3 = q - p + m_3 + {}^1/_2$$
$$\beta_3 = -q - p + n_3 + \begin{cases} 1, & \delta \geqslant \mu(2+\delta) \\ 0, & \delta < \mu(2+\delta) \end{cases}$$
$$a_1 = -\mu c_1, \qquad a_2 = -\delta c_2, \qquad b_1 = d_1, \qquad b_2 = 0 \tag{2.6}$$
$$a_3 = 2i\mu c_3 / (a + \nu - 2i\mu / \delta), \qquad b_3 = -2i\mu d_3 / (a - \nu + 2i\mu / \delta)$$
$$2\pi p = \operatorname{arc\,tg}\frac{1+\mu}{\nu}, \qquad 2\pi q = \operatorname{arc\,tg}\frac{\delta - \mu(2+\delta)}{a\delta}$$
$$a = [\nu^2 + 4\mu(\delta-\mu)(1+\delta)/\delta^2]^{1/2}, \qquad |\,|\operatorname{arc\,tg}\ldots| < \pi/2$$

where $m_1, n_1, \ldots, n_3$ are, so far undetermined, integers.

The integrability (in the conventional meaning) of functions $\Omega / \sqrt{\zeta - \sigma}$ and $Z/\sqrt{\zeta - \sigma}$ in each neighborhood of singular points ζ_k implies that $\alpha_1, \beta_1, \alpha_2, \beta_2 \geqslant -1$, and $\alpha_3, \beta_3 \geqslant {}^1/_2$. By virtue of (2.6) this means that $m_1 > 0$, $m_2 \geqslant 0$, $n_1 \geqslant -1$, $n_2 \geqslant -1$, $m_3 \geqslant 1$, $n_3 \geqslant 0$ for $\delta \geqslant \mu(2+\delta)$ and $n_3 \geqslant 1$ for $\delta <$

$\mu\,(2+\delta)$. The hodograph shown in Fig. 3 (for the case in which (1.2) is satisfied) provides further inequalities. Thus from (2.5) and (2.6) follows that for $\zeta = 0$

$$w(0) = \lim_{\zeta\to 0} i\,\frac{d_1 - c_1\mu\zeta^{\alpha_1-\beta_1}\Phi_{11}/\Phi_{21}}{d_1 + c_1\zeta^{\alpha_1-\beta_1}\Phi_{11}/\Phi_{21}}$$

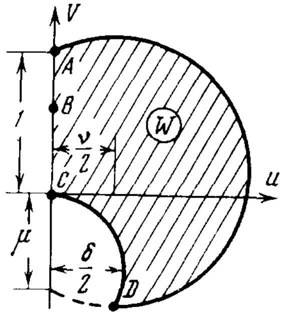

Fig. 3

Hence $w(0) = i$ only for $\alpha_1 > \beta_1$. Similarly we obtain that $\alpha_2 > \beta_2$ and $\alpha_3 > \beta_3$, i.e. $m_1 \geqslant n_1 + 1$, $m_2 \geqslant n_2 + 1$, $m_3 \geqslant n_3 + 1$ for $\delta \geqslant \mu\,(2+\delta)$ and $m_3 \geqslant n_3$ for $\delta < \mu\,(2+\delta)$. The substitution of α_k and β_k from (2.6) into condition (2.4) reduces the latter to the equation in integral numbers

$$m_1 + n_1 + m_2 + n_2 + m_3 + n_3 + \begin{cases} 1, & \delta \geqslant \mu\,(2+\delta) \\ 0, & \delta < \mu\,(2+\delta) \end{cases} = 0$$

It follows from this that $m_1 = m_2 = 0$, $n_1 = n_2 = -1$, $m_3 = 1$, $n_3 = 0$ for $\delta \geqslant \mu\,(2+\delta)$, and $n_3 = 1$ for $\delta < \mu\,(2+\delta)$. Hence

$$\alpha_1 = \alpha_2 = 0, \quad \beta_1 = 1-\gamma = 2p-1, \quad \beta_2 = -1/2 \qquad (2.7)$$
$$\alpha_3 = \alpha = q - p + 3/2, \qquad \beta_3 = \beta = 1 - q - p$$

It was shown in [3] that fundamental solutions of Eq. (2.3) can be presented as follows:

$$\begin{aligned}
\Phi_1^1 &= F(\alpha, \beta; \gamma; \zeta) \\
\Phi_2^1 &= \zeta^{1-\gamma}F(\alpha+1-\gamma, \beta+1-\gamma; 2-\gamma; \zeta) \\
\Phi_1^2 &= F(\alpha, \beta; \alpha+\beta+1-\gamma; 1-\zeta) \qquad (2.8) \\
\Phi_2^2 &= (1-\zeta)^{\gamma-\alpha-\beta}F(\gamma-\alpha, \gamma-\beta; \gamma+1-\alpha-\beta; 1-\zeta) \\
\Phi_1^3 &= (-\zeta)^{-\alpha}F(\alpha, \alpha+1-\gamma; \alpha+1-\beta; 1/\zeta) \\
\Phi_2^3 &= (-\zeta)^{-\beta}F(\beta+1-\gamma, \beta; \beta+1-\alpha; 1/\zeta)
\end{aligned}$$

where $F(\alpha, \beta; \gamma; \zeta)$ is a hypergeometric series convergent within a unit circle, and ζ^λ (λ is real) denotes the branch which is continuous in R_ζ^+ and is determined by the equality $\zeta^\lambda = |\zeta|^\lambda\, e^{i\lambda\arg\zeta}$, where $\arg\zeta$ lies between $-\pi$ and π (the section along the negative semiaxis).

The determination of coefficients $a_k, \ldots, d_k$ requires first of all that Ω and Z satisfy boundary conditions (2.1), which implies that the coefficients c_1, d_1, c_2, d_2, $a_3e^{i\pi\alpha}$ and $b_3e^{i\pi\beta}$ must be real (we use here (2.6)). At the same time any two local representations of function Ω (or Z) must be the same in the common region of their determination. If, for example, we take the expressions for Ω and Z from (2.5) which are valid only within the circle $|\zeta| < 1$ ($k = 1$) and extend Φ_1^1 and Φ^1_2 with the use of known formulas [3] to the inside of circle $|\zeta - 1| < 1$, we obtain for Ω and Z new representations in that circle. However the latter for $k = 2$ must be the same as (2.5), which, by virtue of the linear independence of Φ_1^2 and Φ_2^2, is equivalent to the congruence of coefficients at each of Φ_1^2 and Φ_2^2. This leads to equations relating $a_1, \ldots, d_1$ to $a_2, \ldots, d_2$. The relationships between $a_2, \ldots, d_2$ and $a_3, \ldots, d_3$ are similarly derived. Omitting these somewhat cumbersome calculations, we only note the following results:

$$a_1 = -\lambda\mu, \quad b_1 = \lambda\mu \frac{\Gamma(\gamma)\Gamma(\alpha+1-\gamma)\Gamma(\beta+1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(2-\gamma)}$$

$$c_1 = \lambda, \qquad d_1 = b_1$$

$$b_2 = 0, \qquad c_2 = \lambda \frac{1+\mu}{1+\delta} \frac{\Gamma(\gamma)\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}$$

$$a_2 = -\delta c_2, \quad d_2 = \lambda(1+\mu) \frac{\Gamma(\gamma)\Gamma(\alpha+\beta-\gamma)}{\Gamma(\alpha)\Gamma(\beta)}$$

$$a_3 = -\delta c_2 \frac{\Gamma(\alpha+\beta+1-\gamma)\Gamma(\beta-\alpha)}{\Gamma(\beta+1-\gamma)\Gamma(\beta)} e^{-i\pi\alpha} \tag{2.9}$$

$$b_3 = -\delta c_2 \frac{\Gamma(\alpha+\beta+1-\gamma)\Gamma(\alpha-\beta)}{\Gamma(\alpha+1-\gamma)\Gamma(\alpha)} e^{-i\pi\beta}$$

$$c_3 = a_3 (\nu + a - 2i\mu \ /\delta) \ / \ 2i\mu$$

$$d_3 = b_3 (\nu - a + 2i\mu \ / \ \delta) \ / \ 2i\mu$$

where $\Gamma(\alpha)$ is a gamma function and λ , so far arbitrary, a parameter.

Thus the solution of the boundary value problem (2.1) is of the form

$$\Omega = \lambda\Omega_0, \qquad Z = \lambda Z_0 \tag{2.10}$$

with Ω_0 and Z_0 from (2.5), $\Phi_1{}^k$ and $\Phi_2{}^k$ from (2.8), and $a_k, \ldots, d_k$ from (2.9), when $\lambda = 1$ is set in the latter.

3. Free surface, separation boundary, and parameters of image. By virtue of (1.3) in the neighborhood of the singular point ζ_k we have

$$\omega = \omega_k + \lambda \int_{\zeta_k}^{\zeta} \frac{\Omega_0 \, ds}{\sqrt{s-\sigma}}, \qquad z = z_k - i\lambda \int_{\zeta_k}^{\zeta} \frac{Z_0 \, ds}{\sqrt{s-\sigma}}$$

$$\omega_1 = i\,[\nu T + \mu (L - h)], \quad z_1 = h$$

$$\omega_2 = \delta_1 - \delta R, \quad z_2 = -iR$$

$$\omega_3 = T, \qquad z_3 = L - iT$$

The ω and z determined by these formulas are solutions of the considered problem then and only then, when any two "adjacent" representations for ω (or z) are the same at their common point and conditions (1.1) are satisfied. It can be shown that these requirements are equivalent to the set of the following two systems of equations:

$$h = \lambda \int_0^{\sigma} \frac{\operatorname{Re} Z_0{}^+ \, ds}{\sqrt{\sigma - s}}, \quad T = -\lambda \int_{-\infty}^{0} \frac{\operatorname{Im} Z_0{}^+ \, ds}{\sqrt{\sigma - s}} \tag{3.1}$$

$$L = \lambda \int_1^{+\infty} \frac{\operatorname{Im} Z_0{}^+ \, ds}{\sqrt{s-\sigma}}, \quad R = \lambda \int_{\sigma}^{1} \frac{\operatorname{Re} Z_0{}^+ \, ds}{\sqrt{s-\sigma}}$$

$$\nu T + \mu (L - h) = \lambda \int_0^{\sigma} \frac{\operatorname{Re} \Omega_0{}^+ \, ds}{\sqrt{\sigma - s}}, \quad \delta_1 - \delta R = \lambda \int_{\sigma}^{1} \frac{\operatorname{Re} \Omega_0{}^+ \, ds}{\sqrt{s-\sigma}}$$

$$T-R=\lambda\int_1^{+\infty}\frac{\operatorname{Re} Z_0^+\,ds}{\sqrt{s-\sigma}},\qquad L-h=\lambda\int_{-\infty}^{0}\frac{\operatorname{Re} Z_0^+\,ds}{\sqrt{\sigma-s}} \tag{3.2}$$

As stated in Sect. 1 the channel width $2h$ and the maximum distance T of the free surface from the axis $y = 0$ are specified. Hence the first two equations of system (3.1) can be considered as defining parameters σ and λ, , and the last two the characteristic dimensions L and R of the lens. It appears that system (3.2) is satisfied for σ, λ, h, L and R determined in this manner. In fact, functions $\Omega_0/\sqrt{\zeta-\sigma}$ and $Z_0/\sqrt{\zeta-\sigma}$ derived in Sect. 2 are regular and integrable at singular points ζ_k. Hence the integrals of each of these taken along the real axis are zero, i. e.

$$0=\int_\sigma^{+\infty}\frac{\operatorname{Re}\Omega_0^+\,ds}{\sqrt{s-\sigma}}+\int_{-\infty}^{0}\frac{\operatorname{Im}\Omega_0^+\,ds}{\sqrt{\sigma-s}}=\int_{-\infty}^{\sigma}\frac{\operatorname{Re}\Omega_0^+\,ds}{\sqrt{\sigma-s}}=$$

$$\int_\sigma^{+\infty}\frac{\operatorname{Re} Z_0^+\,ds}{\sqrt{s-\sigma}}+\int_{-\infty}^{0}\frac{\operatorname{Im} Z_0^+\,ds}{\sqrt{\sigma-s}}=\int_1^{+\infty}\frac{\operatorname{Im} Z_0^+\,ds}{\sqrt{s-\sigma}}-\int_{-\infty}^{\sigma}\frac{\operatorname{Re} Z_0^+\,ds}{\sqrt{\sigma-s}}$$

This implies, in particular, that

$$L-h-\lambda\int_{-\infty}^{0}\frac{\operatorname{Re} Z_0^+\,ds}{\sqrt{\sigma-s}}=\lambda\left[\int_1^{+\infty}\frac{\operatorname{Im} Z_0^+\,ds}{\sqrt{s-\sigma}}-\int_{-\infty}^{\sigma}\frac{\operatorname{Re} Z_0^+\,ds}{\sqrt{\sigma-s}}\right]=0$$

which is equivalent to the fourth equation of system (3.2) (the remaining equations are verified in a similar manner).

We represent the equation for determining parameter σ in the form

$$\frac{h}{T}=F(\sigma)\equiv\int_0^{\sigma}\frac{\operatorname{Re} Z_0^+\,ds}{\sqrt{\sigma-s}}\Bigg/\left[-\int_{-\infty}^{0}\frac{\operatorname{Im} Z_0^+\,ds}{\sqrt{\sigma-s}}\right] \tag{3.3}$$

With the use of the inequality $|q| \leqslant p$ we can show that the numerator and the denominator in the formula for $F(\sigma)$ are positive. It follows from the condition $1.5 < \gamma < 2$ [4] that $F(+0) = \nu/(1+\mu)$ and $F(1-0) = +\infty$. Hence for any positive values of parameters h and T such that

$$h/T > \nu/(1+\mu) \tag{3.4}$$

Eq. (3.3) is solvable along the segment $0 < \sigma < 1$. Denoting by Q the over-all discharge rate from the free surface Γ_1 $(QK^{-1} = \nu T + \mu(L-h))$, we can write condition (3.4) in the form

$$Q < Kh + K\mu L$$

This means that the rate of discharge from the free surface owing to evaporation does not exceed (under stabilized conditions) the rate of discharge from the channel in the case of free filtration. The free surface is defined by

$$x(\xi)=L-\lambda\int_{-\infty}^{\xi}\frac{\operatorname{Re} Z_0^+\,ds}{\sqrt{\sigma-s}},\quad y(\xi)=-T-\lambda\int_{-\infty}^{\xi}\frac{\operatorname{Im} Z_0^+\,ds}{\sqrt{\sigma-s}}$$
$$(-\infty<\xi<-1)$$

$$x(\xi)=h-\lambda\int_0^{\xi}\frac{\operatorname{Re} Z_0^+\,ds}{\sqrt{\sigma-s}},\quad y(\xi)=-\lambda\int_0^{\xi}\frac{\operatorname{Im} Z_0^+\,ds}{\sqrt{\sigma-s}} \tag{3.5}$$
$$(-1<\xi<0)$$

It can be shown that $dy / d\xi > 0$, and this gives meaning to the derived formula for Γ_1. In the neighborhood of A ($\xi < 0$ is close to zero) $dx / dy = \mathrm{ctg}\,(2\pi p) - (-\xi)^{\gamma-1}\mu_1\Phi_{11} / \Phi_{21}$ ($\mu_1 > 0$ is a constant), therefore at the limit

$$dx / dy\,|_A = \nu / (1 + \mu) > 0$$

At the same time, if ξ is sufficiently small, $dx / d\xi > 0$, i.e. $x = x(y)$ increases locally strictly monotonically, which leads to the "depression" of the free surface [1]. Calculations show that this peculiarity, which is a shortcoming of the considered model, for real parameters appears only in a very small neighborhood of point A. The dividing boundary is of the form

$$x(\xi) = L - \lambda \int_{\xi}^{+\infty} \frac{\mathrm{Im}\, Z_1^+\, ds}{\sqrt{s-\sigma}}, \quad y(\xi) = -T + \lambda \int_{\xi}^{+\infty} \frac{\mathrm{Re}\, Z^+\, ds}{\sqrt{s-\sigma}} \qquad (3.6)$$

$$(1 < \xi < +\infty)$$

It can be shown that $dx / d\xi > 0$. Numerical calculations indicate a monotonic increase of $y = y(x)$.

4. Calculation results. The dependence of the lens chareacteristic dimensions L and R on parameters μ and ν are shown in Fig. 4, where the upper pair of curves represent $L = L(\mu)$ and the lower $R = R(\mu)$. for $h = T = 1$ and $\delta = 0.1$. Curves *1* and *2* relate, respectively, to $\nu = 0.01$ and 0.1. These curves indicate that the lens dimensions L and R depend to a greater extent on μ than on ν. A similar behavior is also characteristic of particular models, i.e. $\mu = 0$ or $\nu = 0$ [1].

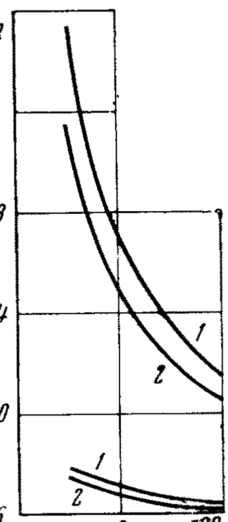

Fig. 4

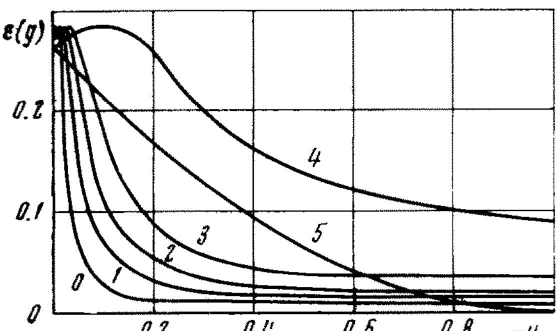

Fig. 5

For $h = 5$, $T = 1$, $\delta = 0.1$, $\nu = 0.281$ and $\mu = 0$, $L = 39.355$ and $R = 9.56$ were obtained in [1]. If in the considered here formulation the same values are specified for h, T, δ and ν, and μ successively diminished ($\mu = 0.01$, 0.005, 0.001, 0.0005, and so on), L and R will tend to their limits: $L = 23.05, 27.41, 35.43, 37.06, \ldots$ and $R = 9.07, 9.26, 9.52, 9.57, \ldots$ A similar agreement with the results obtained in [1] are observed for $\nu \to 0$.

The particular models ($\mu = 0$ or $\nu = 0$) suffer from the shortcoming in that the law of variation of evaporation intensity with depth does not conform to the real one [1] which is fairly accurately approximated by the formula proposed in [5]

$$\varepsilon^\circ(y) = \varepsilon_0 (1 + y / y_0)^n \tag{4.1}$$

where $K\varepsilon^\circ ds$ is the amount evaporated from the element ds of the free surface arc (s is measured from point A), $K\varepsilon_0$ is the intensity of evaporation from the earth surface, and y_0 is the so-called critical depth beyond which evaporation is assumed to be zero. The exponent n is assumed to be between 1 and 3. In the considered formulation the evaporation intensity ε from the free surface Γ_1 is defined by

$$\varepsilon(s) = \mu \, dx / ds - \nu \, dy / ds \tag{4.2}$$

Results of calculations by this formula are given in Fig. 5, where curves $1-4$ define the behavior of $\varepsilon = \varepsilon(y)$ for $\mu = 0.005$, 0.01, 0.02 and 0.08, respectively. Curve (5) calculated by (4.1) for $n = 2$ and $y_0 = T$ is shown there for comparison; $T = 1$, $h = 5$, $\delta = 0.1$ and $\nu = 0.281$ were assumed in this case. The comparison of these curves with the curve taken from [1] and denoted in Fig. 5 by zero shows that the simultaneous introduction of parameters μ and ν makes it on the whole possible to approximate more accurately formula (4.1).

It follows from (4.2) that for $s = 0$

$$\varepsilon(0) = \nu / \sqrt{\nu^2 + (1 + \mu)^2}$$

Owing to the weak dependence of $\varepsilon(0)$ on μ (μ is small), we assumed in calculations that $\varepsilon_0 = \nu / \sqrt{1 + \nu^2}$, and parameter μ was varied so as to obtain an acceptable agreement between formulas (4.2) and (4.1). The curves in Fig. 5 show that the assumptions (1.2) are fairly reasonable, since even for $\mu = 0.08$ curve $\varepsilon(y)$ determined by (4.2) lies as a whole considerably higher than curve $\varepsilon^\circ(y)$ determined by (4.1).

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